Common Core Math in Kindergarten

The main focus in kindergarten is very basic number sense. Of course they'll work on counting as part of this. One aspect that will be new for some classrooms is counting starting from numbers other than one. This helps with addition and subtraction later.

Kindergarteners will compare groups of things to decide which is bigger. They will combine groups together or take some away from a group. Eventually they'll use written numbers to describe what's going on.

Kindergarteners will usually have "rug time" discussion of math as well as play games. A change (for some) is that all of this investigation is carefully directed to develop skills important for later grades.

One of the most important skills in math that students begin in kindergarten is putting things together and taking them apart in various ways. They'll think about different ways that a number can be made from two other numbers as they begin to think about addition and subtraction. The geometry kindergarteners learn reinforces this idea of putting together and taking apart, too. For example, students may be asked to make two triangles from a square or to put together shapes to form a new one.

Examples:

The ideas in "My Book of Five" (see reverse) help kids understand what it means to add and subtract. An important application of this idea comes in representing the "teen" numbers as ten and some more ones (So that 13 means 10 and 3 more ones) because it is the foundation for regrouping/ exchanging (what most of us learned as "borrowing and carrying"). Recognizing the various combinations of numbers that "make up" the numbers from 1 to 10 is a critical building block in learning multi-digit arithmetic.

Tips for parents:

Even though you may not have been taught math in this way, you can still help your child.

- If you count with them, work on starting from any given number.
- Play games that encourage breaking apart numbers in different ways.
- For teen numbers, you may even count in the unit-form way that emphasizes the ten (e.g. eight, nine, ten, ten-and-one, ten-and-two, ten-and-three, ...) as well as with standard names.

Provided by Tricia Bevans and Dev Sinha of the
University of Oregon Department of Mathematics
October 2014

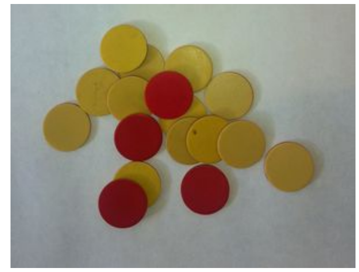
For further information see:
ime.math.arizona.edu/progressions
and illustrativemathematics.org

Example: My book of five

http://www.illustrativemathematics.org/illustrations/1408

Materials:

- Double sided counters
- Markers that are the same colors as the counters
- Teacher-made “My Book of 5” (see below for detailed directions)

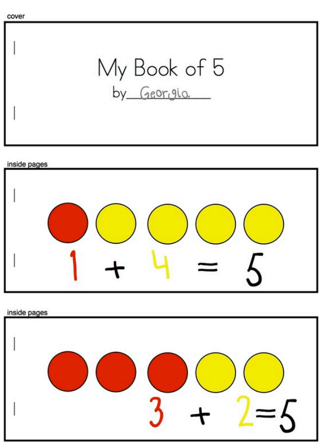


Action:

Students will be given double sided counters/dots (see picture of counters at right). It is important for the markers to match the colors on the counters.

Students take five counters in their cupped hands (or a cup), shake them around, and pour them onto the desk. Next, they count how many counters are yellow and how many are red. Students then record the numbers in their book and write a corresponding equation. For example, if the counters landed so that 1 was yellow and 4 were red, then the student would draw one yellow dot and four red dots and then write “1+4=5” under the drawing. The student would then collect the counters and roll them again. For each combination of colors, the students record with a picture and an equation. Students continue until they fill their book of 5. The teacher can choose how many pages to put in, somewhere between five and eight is a good number so that students get a chance to see multiple combinations.

After the students have completed their books, the teacher should have a whole-group discussion to make the number relationships explicit. One way to do this is to write each of the two addends into a table and to discuss possible patterns and reasons for the pattern. The teacher can ask specific questions such as, “What do you notice about the numbers in the table?” Or “Why is it that as one number gets bigger, the other number gets smaller?”



Common Core Math in 1st Grade

The main ideas in first grade are addition and subtraction up to twenty, and starting to make larger numbers out of tens and ones. In the Common Core, kids will not only learn their number facts, but see them as related. This will help them not only learn these facts, but to build number sense.

For example, a child might learn their "doubles" — such as 8 + 8 = 16 — and from there know close facts such as 8 + 7 = 15 because it must be one less than 8 + 8. Another child might prefer to see 8 + 7 as 8 + 2 + 5, and then see that as 10 + 5 to get 15. This last approach of "making a ten" is key. Finding it this way will help kids remember it and will also be important for knowing the rules of arithmetic and eventually algebra.

Kids will be working in concrete ways with tens and ones — often with blocks, definitely with pictures — so that they know what it means make a ten or break one up. This process is called "regrouping" (we have called it carrying or borrowing in the past, but are we really "borrowing" if we never get it back?) to emphasize that the value of the number hasn't changed. Eventually kids will be proficient with pencil-and-paper and even mental math, but using pictures or objects gives them a firm foundation for what they're doing.

Another small but important change is that kids won't just see problems like 3 + 2 = 5 but also 5 = 3 + 2 and even 3 + 2 = 1 + 4. Well-established research suggests the importance of activities like this to lay a proper understanding of the equal sign.

Examples:

The game "Kiri's Mathematics Matching Game" (see reverse) is like Memory, though one can even start with all cards face up as one is learning the game. The idea is to look for two numbers which add **or** subtract to give a target. So if the target is 6 and you turn over a 4 first you can look for 2 next, because 2 + 4 = 6, or 10 because 10 - 4 = 6. To figure out what you need to turn over, you can use the relationship between addition and subtraction, which is what the game is really about.

And far from the Common Core being "one size fits all," this shows that even kids (in this case a 4J kid) can help create Common Core materials!

Tips for parents:

Here are some ideas for reinforcing the math at home.

- Talking about arithmetic out loud as it comes up in daily life is wonderful.
  "There are six of us at dinner and two cups already out; how many more cups do we need?"
  If you use cash, talking through money is terrific.
- There are many good games that promote good number sense, without your kids even noticing.
  For example, play the card game "War" but use two cards instead — so your 5 + 3 = 8 beats my 2 + 5 = 7. Even better than just doing the addition in this case is reasoning that 5+3 wins because both have 5's but the three is greater than the 2.
- If you want to give kids skill practice, it is better to have activities that encourage reflection. A website or worksheet which has kids do a "plus two" right next to a "plus three" will encourage them to make connections that reinforce recall.

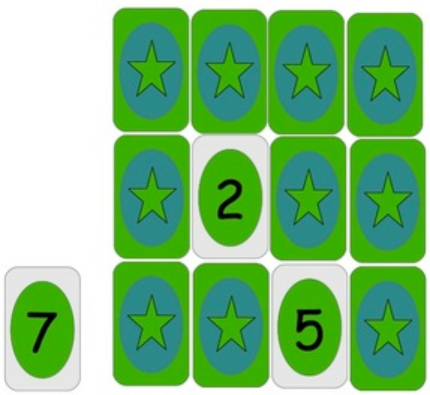
Provided by Tricia Bevans and Dev Sinha of the
University of Oregon Department of Mathematics
October 2014

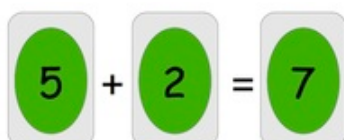
For further information see:
ime.math.arizona.edu/progressions
and illustrativemathematics.org

Example: Kiri's Mathematics Matching Game

http://www.illustrativemathematics.org/illustrations/991

- Students can play in groups of 2–4.
- An array of cards (12–20 in total) is placed face down and one card, called the target card, is put face up.
- The students take turns flipping over two cards, one at a time.
- If the sum or difference of the values on the two cards equals the value on the target card, the student who exposed those cards should say a number sentence to express the relationship. If they are correct, the three cards are removed and replaced so there is again a full array.
- If a student does not combine the values of flipped cards to make the value on the target card, then it is the next student's turn.
- In the no-memory-needed version of the game, all chosen cards are left face up (after an unsuccessful turn) and may be used to make matches. In the light-memory version, cards are left face up until there is a match, after which all are put face down. In the memory version, cards are put face down after an unsuccessful turn before the next player's turn.





In all versions, students must engage basic addition and subtraction facts. In the memory version, after a student has turned over one card, in order to know whether there is a match using cards they've seen, they need to solve equations of the form

$\square+b=c$ $b+\square=c$ $\square-b=c$ and $b-\square=c$.

Students could also be asked to record the number sentences they make.
Teachers could make cards, or have students make them, or use numbered cards from a standard deck or by taking cards from other games. Zeros would be appropriate, and "wilds" could also naturally be incorporated. The target card values should be up to 20 to fully meet the standard (with target cards kept separately).
To extend, and incorporate Standard 1.OA.7 into this activity, there could be two target cards to match in total or difference and/or students could flip over three cards and possibly use all of them.

Note: This game was invented by Kiri, when she was a first grader (now is a fourth grader).

Common Core Math in 2nd Grade

Second graders will continue their work understanding the way our number system works using place values of ones, tens, hundreds, etc. They'll recognize that the 3 in the number 357 represents 3 hundreds rather than “just being a three” and that 12 tens is the same as 1 hundred and 2 tens. Later this will make it clear that adding two hundred to 357 is just a matter of adding 2 to the 3 in the hundreds place.

Kids will work on skip counting by various numbers including tens and hundreds both to increase skill for addition and subtraction using these place values but also as a foundation for multiplication.

While second graders will continue to use many different strategies for adding and subtracting, they use their understanding of the way numbers are built to move toward methods that will always work quickly and accurately.

Geometric concepts they're studying at the same time reinforce the number sense they're working on, provide real world contexts, and give a good foundation for understanding more advanced concepts. For instance, you'll notice that students work with measuring lengths. They might add two different lengths together or compare the lengths of two objects (which would require subtraction). Using bar graphs, clocks, or money they might practice these same skills. In second grade they also do things like partition rectangles into squares and other equal shapes in preparation for understanding both multiplication and fractions.

Examples:

Bundling and Unbundling https://www.illustrativemathematics.org/illustrations/144 (see reverse)

The first part of this task is straightforward, but in part B of this task, the kids have to think a bit more. They're breaking apart the number 14 tens into 10 tens and 4 tens. Then, they recognize that the group of 10 tens can be “bundled” into a group of 1 hundred. This is just what they will need to understand in order to add something like 152 + 91 using the standard algorithm where we line up the ones and the tens and the hundreds and add in columns. Adding 2 and 1 in the ones place is straightforward, but when they add the 5 and 9 in the tens place, the 14 they get will have to be regrouped (or “carried”).

Tips for parents:

- Practice in everyday situations. For example, ask your child to compare the price of two different items and decide how much you would save. Count by 2's, 3's, 4's, etc. to figure out how many there are of something rather than counting one at a time.

- You may find that there are methods of writing basic arithmetic that are unfamiliar to you. Often, these are just ways of recording more of the thinking that goes into the math. Try to understand the process yourself, checking in with the teacher if need be. If you do want to share the way you learned make sure you can also explain the thinking around it as well as how it relates to the ways things are being done in class.

- Have your child explain how she found an answer using words or pictures, sometimes even if the process is easy for her.

Example: Bundling and Unbundling

https://www.illustrativemathematics.org/illustrations/144

Make true equations. Write one number in every space. Draw a picture if it helps.

a. 1 hundred + 4 tens = _____ ; 4 tens + 1 hundred = _____
b. 14 tens = 10 tens + ____ tens ; 14 tens = ____ hundred + 4 tens ; 14 tens = ____ ones
c. 7 ones + 5 hundreds = _____
d. 8 hundreds = _____
e. 106 = 1 hundred + ____tens + ____ones ; 106 = ____tens + ____ones ; 106 = ____ones
f. 90 + 300 + 4 = _____

Commentary:

Students determine the number of hundreds, tens and ones that are necessary to write equations when some digits are provided. Student must, in some cases, decompose hundreds to tens and tens to ones. The order of the summands does not always correspond to the place value, making these problems less routine than they might be.

Solutions:

a. 140, 140
The first problem asks for the same number (140) in different ways. This emphasizes that order doesn't matter in addition — yet order is everything when using place-value notation.

b. 14 tens = 10 tens + 4 tens 14 tens = 1 hundred+4 tens 14 tens = 140
In this problem, the base-ten units in 140 are bundled in different ways. In the first line, "tens" are thought of as units: 14 things = 10 things + 4 things.

c. 507
By scrambling the usual order, the third problem requires students to link the values of the parts with the order of the digits in the positional system. Also, to encode the quantity, the student will have to think: "no tens," emphasizing the role of 0. 7 ones + 5 hundreds = 507.

d. 800
In the fourth problem, the zeros come with a silent "no tens and no ones": 8 hundreds = 800.

e. 106 = 1 hundred + 0 tens + 6 ones 106 = 10 tens + 6 ones 106 = 106 ones
In this problem, the base-ten units in 106 are bundled in different ways. This is helpful when learning how to subtract in a problem like 106 – 34 by thinking about 106 as 100 tens and 6 ones.

f. 394
The sixth problem is meant to illustrate the notion that if the order is always given "correctly," then all we do is teach students rote strategies without thinking about the size of the units or how to encode them in positional notation. 90 + 300 + 4 = 394.

Common Core Math in 3rd Grade

Back in the old days, third grade math was all about multiplication. In the Common Core, that's what it is still about!

A key change is that now we want students to apply their multiplication skills to more story problems, as well as connect the multiplication facts to one another. For example, if a child knows their "times fours," that can be used to help recall or figure out their "times eights": Since $3 \times 4 = 12$, then 3×8 must be twice that or 24. Some but not all kids have used these kinds of strategies in the past. Now they will be used widely, and they will all be discussed so the kids who notice these kinds of things learn to not just see it but describe what and why.

Kids will see pictures explaining these connections (see example below). Students will also do multiplication and division together more, rather than seeing them separately. So, for example, soon after students learn that $4 \times 6 = 24$ they'll learn it also means that $24 \div 4 = 6$ and $24 \div 6 = 4$. Kids will also be mastering addition and subtraction in the hundreds. This will mean not only learning the standard way, but figuring out short cuts and alternate approaches and talking about why they work. For many reasons we'd like to see kids see an addition such as $398 + 15$ and not have to "line it up" to add but instead say, "Well, if we give two of the 15 to the 398 that makes 400 so the answer is 413," or, "If we look on the number line, only two steps are needed to get to 400, and then 13 steps more would be 413."

Examples:

Eureka Math: Demonstrating the Commutativity of Multiplication (see reverse)
https://www.engageny.org/resource/grade-3-mathematics-module-1

Here we see third graders using pictures of neatly organized objects called rectangular arrays (or just arrays). In the Common Core, students will begin to use arrays in second grade, so they will already be familiar.

In this worksheet, students use these arrays to see why we get the same amount when we calculate $2\ x\ 6$, (that is, two sixes) and 6×2 (that is, six twos). Later they fill in $2 \times 9 = 9 \times$ _____. Here, instead of having two problems to evaluate and get the answer of eighteen, students see these as directly related. This is emphasizing how arithmetic follows rules which eventually become the rules of algebra.

Tips for parents:

- If you practice multiplication facts, try to highlight related facts especially when your child cannot recall one. For example, if they don't remember 6×6 right away, you can ask, "Do you remember 5×6?" If they do, then remind them (if needed) that 6×6 is just six more.

- Be patient with the rectangular arrays and other unfamiliar approaches. No method is perfect, but for many students and teachers their use has already proven to be more effective than what we were doing in the past.

- It should be fine to show your child the standard "line them up" ways to add and subtract (and they will see them in class too!) but realize that they may need to provide an alternate approach, especially when the standard way isn't as efficient as some meaningful shortcut.

Example: Demonstrating the Commutativity of Multiplication, Eureka Math Module 1 Lesson 7 (excerpt)

1. a. Count by 2 six times.

   ______, ______, ______, ______, ______, ______

   b. Draw an array that matches your count-by.

   c. Write a multiplication sentence that represents the total number of objects in your array.

   ________ × ________ = ________

2. a. Count by 6 two times.

   ________, ________

   b. Draw an array that matches your count-by.

   c. Write a multiplication sentence that represents the total number of objects in your array.

   ________ × ________ = ________

3. a. Compare your work in Problems 1 and 2. Turn your paper as you study the arrays to look at them in different ways.

   b. Why are the factors in your multiplication sentences in a different order?

Write and solve a different multiplication sentence to describe each array.

____

____

Common Core Math in 4th Grade

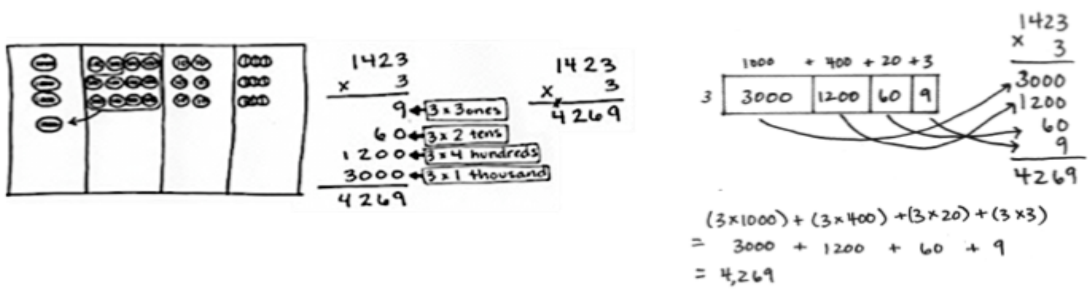
The two most important areas of focus for this grade are skill with multiplication and division and building understanding of fractions.

Fourth graders will develop understanding of and fluency with multi-digit multiplication and division. Eventually, they should be comfortable with methods for multiplication and division that work quickly and accurately. This includes the usual procedures, as well as some which could be faster in some cases or more understandable to the students. For example, 35 x 12 = 35 x 2 x 6 = 70 x 6 = 420. This not only helps when calculator or pencil-and-paper are not available, but helps to prepare for algebra. To be sure these processes work, and to better prepare for algebra, the students will use pictures and other methods to explain why they work.

Working with fractions is another key element of 4th grade math. To understand why fractions have many names for the same number — for example $\frac{1}{2}$ is the same as $\frac{2}{4}$ is the same as $\frac{3}{6}$ and so on — students will use pictures, instead of "canceling," which doesn't really have any meaning for kids at that point, or using fraction multiplication, which would be using an advanced topic for more basic understanding.

Examples:

From Eureka Math: Grade 4 Module 3 Topic C Overview
https://www.engageny.org/resource/grade-4-mathematics-module-3-topic-c-overview



Each of these descriptions of how to calculate 1423 times 3 is useful in different ways. The first uses place value (the meaning of ones, tens, hundreds and thousands) and connects multiplication to addition. The middle two descriptions are expanded and condensed versions of the standard algorithm. The last uses area to represent the multiplication and connects the other descriptions with ideas needed in algebra. Students will learn to see the connections between these methods both to check their work and to reinforce why each process works.

Tips for parents:

- Communicate with your child's teacher if you are regularly unable to help your child with unfamiliar multiplication or division methods.

- Do math in everyday settings. Encourage your child to recognize fraction equivalence in activities like cooking, for example "I can put in one cup and a half cup of milk or three half-cups of milk." There are lots of multiplication and division examples, for example estimating how many candies they'll get from trick-or-treating.

- Especially if your child catches on to procedures quickly, make sure she or he can explain why something makes sense.

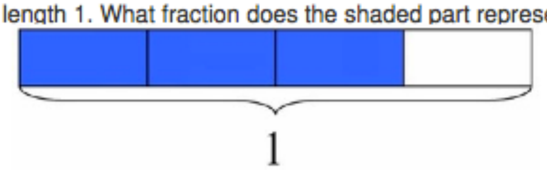
Provided by Tricia Bevans and Dev Sinha of the
University of Oregon Department of Mathematics
October 2014

For further information see:
ime.math.arizona.edu/progressions
and illustrativemathematics.org

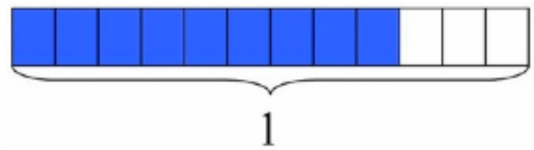
Example: Explaining Fraction Equivalence with Pictures

https://www.illustrativemathematics.org/illustrations/743

a. The rectangle below has length 1. What fraction does the shaded part represent?



b. The rectangle below has the same length as the rectangle above. What fraction does the shaded part represent?



c. Use the pictures to explain why the two fractions represented above are equivalent.

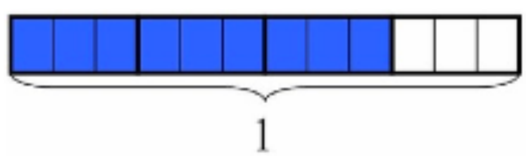
Commentary:

The purpose of this task is to provide students with an opportunity to explain fraction equivalence through visual models in a particular example. Part C should be approached as a discussion before students are asked to write an explanation. Students can talk generally about the relationship between the pictures ("Each of the larger pieces is broken up into 3 little pieces"), which can then be refined and connected to the appropriate operations ("There are three times as many smaller pieces as bigger pieces"). Students will need more opportunities to think about fraction equivalence with different examples and models, but this task represents a good first step.

Solutions:

a) ¾ b) 9/12

c. Three pieces in the bottom rectangle have the same size as 1 piece in the top rectangle. We can even show this by darkening the lines around groups of three small pieces in the rectangle that represents $\frac{9}{12}$:



When we make groups of three in the bottom rectangle, there are 3 groups of 3 shaded pieces and 4 groups of 3 in the whole rectangle. Using these groups, we see that

$$\frac{9}{12} = \frac{(3 \times 3)}{(4 \times 3)} = \frac{3}{4}$$

of the bottom rectangle is shaded. Since the shaded portion is the same in each case but we just look at it in a different way and describe it with a different fraction, the fractions are equal. So

$$\frac{9}{12} = \frac{3}{4}$$

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Common Core Math in 5th Grade

Fifth grade in the Common Core is when students finish having arithmetic as a focus, though in later grades there will be plenty of opportunity to continue practicing these skills — for example, dividing numbers when computing proportions.

This year students will learn to add fractions. This is a complicated process, and some curricula even suggest using elaborate gimmicks to remember it. In the Common Core, students will have a firm grounding in the number line, in renaming fractions (e.g. $\frac{2}{3}$ is also $\frac{4}{6}$) and in adding fractions with the same denominator ($\frac{3}{8} + \frac{2}{8} = \frac{5}{8}$). All of this will make addition of fractions a process that makes sense, rather than something to remember using tricks which use pictures of X's or butterfly wings.

This type of reasoning also helps to *apply* fraction arithmetic correctly. Many of us remember that you "multiply across" to multiply $\frac{2}{3} \times \frac{4}{5}$, but struggle to know if one *should* multiply in a real world context. A key is that $\frac{2}{3} \times \frac{4}{5}$ is what you get when you split $\frac{4}{5}$ of something into three equal pieces and take two of those. Students will use pictures to reason about problems, as many good problem-solvers often do. From these they will be able to know whether to multiply or divide, and have a sense for what a reasonable answer should be.

Students will use similar reasoning about whole numbers and decimals — using sketches, examples, and properties which have been carefully developed, so these arithmetic skills will provide a strong base for algebra.

Examples:

Video Game Scores (see reverse) https://www.illustrativemathematics.org/illustrations/590
In this task, students connect a "real-life" situation to arithmetic with many steps. The students don't have to evaluate the scores, though a teacher could ask them to if necessary. The more important part of the activity is to have students work on their mathematical language skills to interpret expressions in the context of the problem. This gives some great practice leading up to using variables as in algebra. One can just change the task a bit — an unknown amount of bonus points, for example — and it is a good algebra activity.

Tips for parents:

- It is likely that your child is learning in a way you didn't, so you can't just figure out in a minute what's going on. This presents a great opportunity: ask your child to explain some math to you! Communicating reasoning is a skill we want children to have, and it rarely happens enough.
- Kids at this point will likely have a strong sense of how "good" they are at math, usually based on how quickly they can calculate. Challenge this! Many of the best mathematicians are slow at calculation, but take time to truly understand a problem. Understanding will eventually be a struggle for everyone in some math class. Just as a musician doesn't expect to play every new piece well, a math learner won't understand every concept right away but can progress until they get there.
- High achievers may be ready to use variables to more deeply reflect on the arithmetic they learn. If they see exactly why three fourths and two fourths makes five fourths (on the number line, especially), and similarly nine fourths and two fourths makes eleven fourths and so on, then they could also say that *n* fourths and two fourths makes *n* + 2 fourths. In symbols, that's $\frac{n}{4} + \frac{2}{4} = \frac{n+2}{4}$. This deeper reflection on fraction arithmetic is much more beneficial than rushing though the rules of arithmetic on an accelerated track.

Provided by Tricia Bevans and Dev Sinha of the
University of Oregon Department of Mathematics
October 2014

For further information see:
ime.math.arizona.edu/progressions
and illustrativemathematics.org

Example: Video Game Scores

https://www.illustrativemathematics.org/illustrations/590

Eric is playing a video game. At a certain point in the game, he has 31500 points. Then the following events happen, in order: He earns 2450 additional points. He loses 3310 points. The game ends, and his score doubles.

Write an expression for the number of points Eric has at the end of the game. Do not evaluate the expression. The expression should keep track of what happens in each step listed above.

Eric's sister Leila plays the same game. When she is finished playing, her score is given by the expression 3(24500+3610) − 6780. Describe a sequence of events that might have led to Leila earning this score.

Commentary:

Standard 5.OA.2 asks students to "Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them." This task asks students to exercise both of these complementary skills, writing an expression in part A and interpreting a given expression in part B. The numbers given in the problem are deliberately large and "ugly" to discourage students from calculating Eric's and Leila's scores. The focus of this problem is not on numerical answers, but instead on building and interpreting expressions that could be entered in a calculator or communicated to another student.

Solution:

a. When Eric earns 2450 additional points, his score becomes 31500 + 2450. When he loses 3310 points, his score becomes (31500 + 2450) – 3310. (Note that this can also be written without the parentheses.) When Eric's score doubles, the score becomes 2 x ((31500 + 2450) – 3310), which can also be written 2(31500 + 2450 – 3310).

b. Here is a possible sequence of events that might lead to the score given: At a certain point in the game, Leila has 24500 points. She earns 3610 additional points. Her score triples. She loses 6780 points.

c. Note that the order of the steps is important; rearranging the steps will likely lead to a different expression and a different final score.

Common Core Math in 6^{th} Grade

In sixth grade different number and arithmetic concepts come together and are used in interesting ways. Students are going to use their knowledge of multiplication and division to understand problems involving ratios and proportions. They'll increase their skill with fractions to include dividing fractions. And they'll begin to use equations and expressions with variables. Along the way, they'll also fill in the number line with one more type of number as they begin to understand and work with negative numbers.

These topics are all highly interrelated. Students will use tables, graphs, number lines, and diagrams to represent a situation with ratios as different approaches to problem solving and to highlight different structure. For example, suppose a juice blend uses 5 cups of grape juice for every 2 cups of peach juice. A student could use a table to find "easy" combinations of peach and grape juice like 10 cups of grape juice and 4 cups of peach juice, then 15 cups and 6 cups, etc. noticing that each time the amount of grape juice increased by 5 while the peach juice increased by 2. Graphing these pairs on a coordinate plane would show further structure and prompt further insights. The standard approach of "cross multiplying" will be a natural result of a solid understanding of the meaning of ratios in seventh grade. Finding the unit rate for ratios involving fractions will add a further context for fraction division then as well. (For further explanation see http://commoncoretools.files.wordpress.com/2012/02/ccss_progression_rp_67_2011_11_12_corrected.pdf)

Examples:

Security Camera: https://www.illustrativemathematics.org/illustrations/115 (see reverse)

This example gives a sense of how students might get to tie together several different concepts in a single context. For this task students must work with fractions, reason about areas and shapes, calculate percentages — all in a context that has some grounding in the real world. It also highlights the mathematical practices that are so important. This is not a problem that's a breeze-through if you understood the examples in the text. Kids are going to have to do some reasoning. They'll have to stick with it. They'll have to communicate why they know they've found the *best* answer. It is possible for nearly every student to begin working on the problem, but there are many opportunities for pushing kids beyond the original problem if they are ready for that too. (Is putting the cameras at grid lines realistic? Does our answer change if we don't have to do that?)

Tips for parents:

- Be patient if your child struggles, especially if math has been relatively easy in the past. Make sure to emphasize that this struggle is not an indication of failure and mistakes are just opportunities to learn (see Carol Dweck's work on mindset).

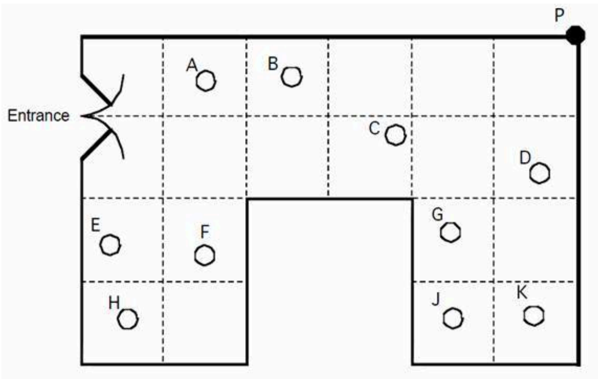
- Continue to have your child practice math as it comes up in your everyday interactions. (e.g. If it has taken us 3 hours to get two thirds of the way to the cabin, how long do you expect the whole trip will take? Will I have enough money to get 2 pairs of pants and 3 shirts?)

- Because middle school begins a transition to more realistic modeling situations, ask your child to notice assumptions you make to solve everyday problems with math. For example, if 6 oz. costs $3.25, how much will 15 oz. cost? Multiplying the cost by 2½ assumes that you *can* purchase 15 oz., and that the unit price is the same for larger quantities.

Example: Security Camera

https://www.illustrativemathematics.org/illustrations/115

A shop owner wants to prevent shoplifting. He decides to install a security camera on the ceiling of his shop. Below is a picture of the shop floor plan with a square grid. The camera can rotate 360°. The shop owner places the camera at point P, in the corner of the shop.



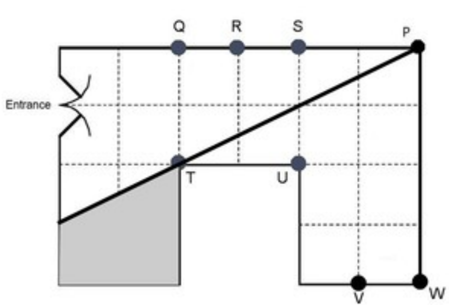
1. The plan shows where ten people are standing in the shop. They are labeled A, B, C, D, E, F, G, H, J, K. Which people cannot be seen by the camera at P?
2. What percentage of the shop is hidden from the camera? Explain or show work.
3. The shopkeeper has to hang the camera at the corners of the grid. Show the best place for the camera so it can see as much of the shop as possible. Explain how you know that this is the best place to put the camera.

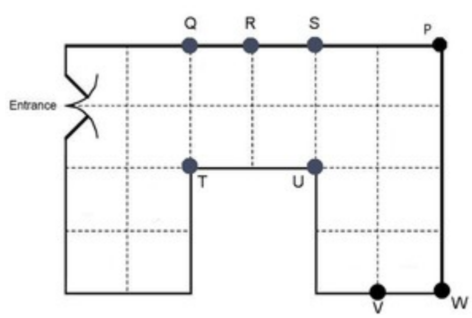
Commentary:

The last question has more than one answer, in the sense that there are three spots that could be considered "best." These three locations all cover the same amount of the store while at the same time miss less of the store than all other possible spots.

Solutions:

1. With the camera at point P, shoppers F and H are hidden from the camera.
2. There are 20 squares on the grid. If a line is drawn from point P to point T and beyond, the region that is hidden from the camera has an area of 3 squares (this region is composed of a triangle with an area of 1 square and a rectangle with an area of 2 squares; see the figure below). There are a total of 17 out of 20 squares visible from point P. 17/20 =0.85, so 85% of the store is visible, and 15% of the store is hidden from point P.
3. Looking at the figure below, the best places to place the camera are Q, R, and S.





Provided by Tricia Bevans and Dev Sinha of the
University of Oregon Department of Mathematics
October 2014

For further information see:
ime.math.arizona.edu/progressions
and illustrativemathematics.org

Common Core Math in 7th Grade

Seventh grade math is some of the most useful throughout life. Calculating discounts, taxes, interest, etc. are something all adults need to do regularly. Now, however, students do more work of recognizing how a percent or proportion comes about and what it means. For example, we can look at a lot of items at a store and ask for each what would be better: a $20 discount or a 20% discount? Letting students figure out that 20% is best for items over $100, and $20 is best for items under $100, from examples (and reason about why) helps them learn about functions later. In fact, one *can* set this up as a function problem, but reasoning directly, perhaps drawing a picture (like the "tape diagrams" borrowed from Singapore), is more intuitive for many.

Learning about negative numbers will also have an emphasis on both context (money owed, temperatures below zero, blocks to the left and right of some landmark) and how previous arithmetic must apply to it. For example, they'll justify why a negative times a negative must be a positive using area calculations of rectangles with negative numbers (e.g. one side is 10 + -3 =7 feet long).

As data is a key part of understanding our world now, this will be a focus. Students will look at two quantities or two populations, and try to understand not only how they are related but how certain they can be about the relationship. Probability at this grade is important in its own right but also reinforces fraction arithmetic.

While a lot of good work happens in seventh grade, the overlap with sixth and eighth grade means that this can be a time for acceleration if needed. Much of the proportional reasoning is similar to sixth grade, so students who are really fluent in fraction arithmetic and proportional reasoning could gain these skills faster. This could allow for a strong precalculus preparation in eleventh grade and calculus in twelfth grade, though calculus in high school isn't recommended as a standard path. Professors prefer students who thoroughly understand algebra and functions to students who have superficial understanding of concepts through calculus.

Examples:

Cooking with the Whole Cup https://www.illustrativemathematics.org/illustrations/470 (see reverse)

Initially one can use "common sense" here: one cup of butter instead of an eighth cup is eight times as much, so he'll need eight times as much of the other ingredients. This reinforces why dividing by an eighth should be the same as multiplying by eight since "how many times does 1/8 cup fit into 1?" is $1 \div \frac{1}{8}$. Later in the problem, "common sense" is no longer enough on its own. Then, knowing how to systematically set things up and think about things like unit rates solves a problem that isn't so easy. By doing an easier case at the beginning, students can check their mathematical process and make sure it agrees with common sense, when both can be applied.

Tips for parents:

- Talk through some good "real world" problems, especially if it takes you a while and you're sharing your thinking. One-on-one discussions about math thinking and reasoning — with a teacher at times, with friends, with you, with a tutor if you have access — is a great experience.
- Your attitude about learning math is crucial. Students should sense that math is worth their attention, and will require effort more than quick thinking or "innate smarts" to be really good at in the long run. The quick thinkers often have trouble once things get more involved, as real-world problems often do.
- Engaging in activities that use math with them is a great way to reinforce both positive attitude and skills — for example, do fraction arithmetic as part of baking or working through financial calculations.

Provided by Tricia Bevans and Dev Sinha of the
University of Oregon Department of Mathematics
October 2014

Cooking with the Whole Cup

http://www.illustrativemathematics.org/illustrations/470

Travis was attempting to make muffins to take to a neighbor who had just moved in down the street. The recipe that he was working with required $\frac{3}{4}$ cup of sugar and $\frac{1}{8}$ cup of butter.

1. Travis accidentally put a whole cup of butter in the mix.
   A. What is the ratio of sugar to butter in the original recipe? What amount of sugar does Travis need to put into the mix to have the same ratio of sugar to butter that the original recipe calls for?
   B. If Travis wants to keep the ratios the same as they are in the original recipe, how will the amounts of all the other ingredients for this new mixture compare to the amounts for a single batch of muffins?
   C. The original recipe called for $\frac{3}{8}$ cup of blueberries. What is the ratio of blueberries to butter in the recipe? How many cups of blueberries are needed in the new enlarged mixture?
2. This got Travis wondering how he could remedy similar mistakes if he were to dump in a single cup of some of the other ingredients. Assume he wants to keep the ratios the same.
   A. How many cups of sugar are needed if a single cup of blueberries is used in the mix?
   B. How many cups of butter are needed if a single cup of sugar is used in the mix?
   C. How many cups of blueberries are needed for each cup of sugar?

Commentary:

While the task as written does not explicitly use the term "unit rate," most of the work students will do amounts to finding unit rates. A recipe context works especially well since there are so many different pair-wise ratios to consider. This task can be modified as needed; depending on the choice of numbers, students are likely to use different strategies which the teacher can then use to help students understand the connection between, for example, making a table and strategically scaling a ratio. The choice of numbers in this task is already somewhat strategic: in part 1, the scale factor is a whole number and in part 2, the scale factors are fractions. Because of this difference, students will likely approach the parts of the task in different ways.

Solutions:

1. A. The ratio of cups of sugar to cups of butter is $\frac{3}{4}:\frac{1}{8}$. If we multiply both numbers in the ratio by 8, we get an equivalent ratio that involves 1 cup of butter. $8 \times \frac{3}{4} = 6$ and $8 \times \frac{1}{8} = 1$ In other words, $\frac{3}{4}:\frac{1}{8}$ is equivalent to 6:1, and so 6 cups of sugar are needed if there is 1 cup of butter.

   B. In the previous part we saw that we have 8 times as much butter, so all the ingredients need to be increased by a factor of 8. That is, the quantity of each ingredient in the original recipe needs to be multiplied by 8 in order for all the ratios to be the same in the new mixture.

   C. The ratio of cups of blueberries to cups of butter is $\frac{3}{8}:\frac{1}{8}$ in the original recipe, so Travis will need to add $8 \times \frac{3}{8} = 3$ cups of blueberries to his new mixture.

2. …C. The ratio of cups of blueberries to cups of sugar is $\frac{3}{8} : \frac{3}{4}$. If we multiply both numbers in the ratio by $\frac{4}{3}$, we get an equivalent ratio. $\frac{4}{3} \times \frac{3}{8} = \frac{1}{2}$ and $\frac{4}{3} \times \frac{3}{4} = 1$. Since $\frac{3}{8}:\frac{3}{4}$ is equivalent to $\frac{1}{2}$: 1, Travis would need $\frac{1}{2}$ cup of blueberries if there is one cup of sugar.

Common Core Math in 8th Grade

Extensive work with linear equations (equations whose graph is a line) ties together much of what your student will learn this year. They'll understand them in the context of functions and represent them using tables, graphs, and equations. They'll take data that suggest a linear relationship, find an appropriate line, and make predictions based on the graph or the equation. Geometry will center around lines as well — shifting, stretching or reflecting 2- and 3-dimensional objects using specific lines as a reference. Linear functions will be one basis for understanding more complicated functions such as quadratic and trigonometric functions, and links to the extensive sixth and seventh grade work with proportional relationships. They will also be analyzing angles formed when lines intersect, and finding the distance between two points on a line using the Pythagorean Theorem.

Examples:

A great activity that introduces the use of linear equations is "Barbie Bungee." Students experiment by measuring how far Barbie falls when using a "bungee cord" made up of a few rubber bands linked together. Then they make predictions about how many rubber bands would be needed for a much higher drop, based on the linear graph that emerges from their data.

One teacher who popularized this is Fawn Nguyen, a middle school teacher in California who shares her classroom activities on a blog. Other teachers, including some in Lane County, have used things like water balloons in place of Barbie. Here's Fawn's description of Barbie Bungee. The overheard student comments are great! http://fawnnguyen.com/barbie-bungee/ (see reverse)

In this Teaching Channel Video students use different methods to find the line of best fit as well as analyzing the linear equation. https://www.teachingchannel.org/videos/stem-lesson-ideas-bungee-jump

An activity like this provides a very concrete context for interpreting the different parts of a linear equation. Why does the line cross the y-axis at this point? Where does that number come from? These are questions that students can address in a natural way here, so that lines to them aren't just "$y = mx + b$".

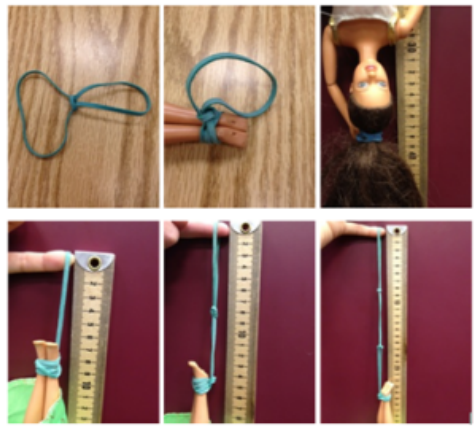
Tips for parents:

- Encourage productive struggle and perseverance. Careful and complete reasoning is much more important than quickly arriving at an answer.
- There are skill-based supports emerging that are aligned with Common Core. The Khan Academy is working hard to create worthwhile tasks, for example. But if your child needs extra support, you might consider working together with your child on an activity like the Barbie Bungee one or activities from Mathalicious, which because they are interesting and sometimes "real world" are more likely to engage them.
- There are many contexts in which linear functions arise, for example costs of cell phone plans as discussed in the high school handout. If you or someone you know can point this out and use them to show why the material is important and how what they are learning in school is a helpful skill, this can help motivation and engagement at a challenging age.

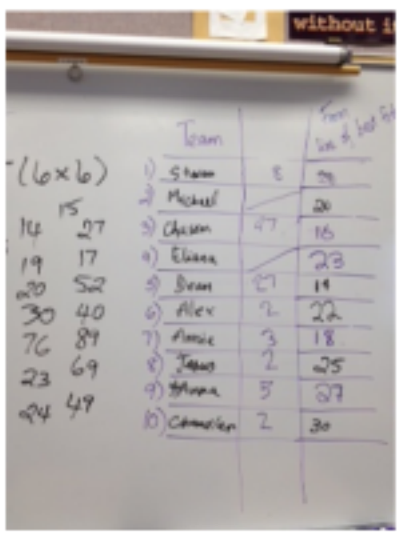
Example: Barbie Bungee

Objective: In teams, create a bungee line for Barbie to allow her the most thrilling, yet SAFE, fall from a height of 3 meters. First measure Barbie's height for up to six rubber band lengths, and record in a table.

Rubber band length	Distance of Fall *cm*
0	28.5 (Barbie's height)
1	35.8
2	43.1
3	
4	
5	
6	



Once groups made their prediction, I drop Barbie. (The numbers on the left were their initial guesses before doing anything else.)



This was a blast!! I had two kids lying on the ground with meter sticks as judges. We clearly had a winning jump when one group's Barbie came within 2 cm of the floor.

What I heard around the room: "I noticed the centimeters went up by 10 on average." "Her height is the y-intercept." "Nine rubber bands is approximately 100 cm, so we need…" "Stop stretching the rubber bands, you're gonna ruin our estimate!" "Each meter stick is 98 cm."(His two teammates did not say anything when they heard this!) "I have to re-do our graph. I stuck it too close to the top, and the line of best fit has nowhere to go." "You're not supposed to connect the dots!" "This was so much fun!" "Oh, I didn't realize how stretchy the rubber bands got." (To which another student said, "Hello, it's rubber.") "Ken is heavier [than Barbie]. We forgot this." "Hair centimeters! She was that close!"

Common Core Math in High School

We will discuss all of high school together, as opposed to one class (Algebra 1 or Integrated Math 2) or one grade at a time. Regardless of names and labels, high school math should primarily be about modeling with algebra and functions. Our discussion here includes examples, which at high school can be sophisticated. Even if you don't follow all of the math, we hope you get a sense for how things are thought through and are designed to better serve your student. Here are some suggestions for what concerned parents might do to help their kids, followed by a discussion of the major shifts in Common Core high school math.

Tips for parents:

- Talk about career choices, and investigate together what math is required for a university or associate degree, a technical certificate, or possible on-the-job needs. Plenty of jobs use math, especially things like proportional reasoning and linear functions, jobs ranging from nursing to forestry to operations to accounting to computer-aided design to carpentry.

- Make sure your kids understand fractions and middle school math — especially proportional reasoning — super well. Work on real-world problems in daily life to reinforce these skills. For example, you can discuss financing their college, or have them imagine what their budget will be when they are 25 years old and discuss financing a car. Too often, in the past, arithmetic skills were lost (because they were based on memorization and hadn't been reinforced) by the time kids got to college.

- Consider using resources such as Mathalicious or Dan Meyer 3-Act Tasks if you want enrichment or extra practice. Khan Academy is working on Common Core skill practice as well.

- Enjoy math! It feels good to put some effort forward and figure something out. Work on your own to model this. Google "Carol Dweck mindset" to understand how important attitudes towards effort and learning are.

Common Core Shift: Applied problems, often based on simpler math

Most math in the world is done to serve some application to science, business or daily life. But we have not taught application of math well! A famous study from the '80s with freshman engineering students showed they had essentially no skill in even setting up equations based on simple situations like "there are six students for each professor."

A look at our old textbooks provides a good explanation: we haven't actually taught applied math. We give students "word problems" exactly like those already worked out in the text. But in real life, when we need to understand a financing plan, no one tells us "look at page 314 of your book to see how to do a very similar problem." We have been denying our kids the opportunity to use the math like we want them to! A TED talk by Dan Meyer (http://www.ted.com/talks/dan_meyer_math_curriculum_makeover) gives one teacher's terrific explanation of how our old textbooks have actually hurt kids' problem solving abilities.

Consider an example with cell phone plans:

> Plan 1: $50/month with unlimited voice and data
> Plan 2: $25/month with $0.10 per minute or MB of data

We can "just figure this out" — the $25 difference up front would pay for 250 minutes of talking or MB of data. So if we think we'll use less than that 250 minutes/MB then the second plan is better, and if we use more the first plan is better.

Now suppose there are three plans:

Plan 1: $50/month with unlimited voice and data
Plan 2: $25/month with $0.10 per minute or MB of data
Plan 3: $35/month with $0.05 per minute or MB of data

"Just figuring it out" becomes complicated. This is the kind of problem where more systematic approaches supported by math are important (see solutions at http://www.illustrativemathematics.org/illustrations/472). Things are better organized using graphs, which show how to find the cheapest plans across different possible minutes/ megabytes.

Interestingly, this task is grounded in eighth grade math, which is when students will first study linear functions. These plans are all perfectly represented by linear functions, since the per-minute rates are constant (in one case zero). In the classroom, students might see this kind of problem first as a whole-class or group project at eighth or ninth grade. They should in later grades be given chances to do these kinds of problems more on their own. In eleventh grade, on the Smarter Balanced assessment, this is the kind of activity that can be part of a "performance task."

By asking students to do these kinds of tasks, we are saying that in addition to some more advanced high school math, authentic application of simple math is important for college and career readiness. Research papers in disciplines like economics often use exactly this kind of "eighth grade" math. More sophisticated math can arise from questions as simple as "why are honeycombs hexagonal?"

Some of the most successful high school teachers have been using these kinds of activities well before the Common Core. Dan Meyer (Google "Dan Meyer three-act tasks"), Fawn Nguyen, a group of teachers with a website called "Mathalicious" (which has some free and some fee-based lessons), and the Mathematics Assessment Project provide many examples. Mathalicious developed an activity called "Text Me Later," for example, in which students time each other texting short messages, and then calculate how far a car travels in that time. These kinds of activities lend themselves to project-based learning, team teaching, and community engagement.

Common Core Shift: Purpose for math skills

A second change that the Common Core prescribes is for skills, especially in algebra, to be applied with a purpose in mind. Not only should skills be applied fluently, but students should recognize *when* and *why* they should be applied. For example, in algebra students have long been asked to simplify quadratic expressions or solve quadratic equations. Now, students may be given the height of a rocket as a function of time,

$h(t) = -16(t^2 - 2t - 5)$ feet

and asked to put it in completed square and factored form as shown below.

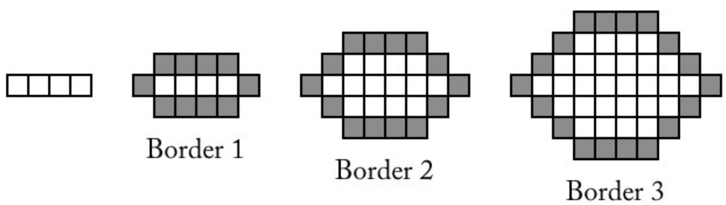
$h(t) = -16(t-1)^2 + 96$ $\quad$ $h(t) = -16\left(t - \left(1 - \sqrt{6}\right)\right)\left(t - \left(1 + \sqrt{6}\right)\right)$

Rather than just an arbitrary skill, this reveals its maximum height and total time of flight. The initial equation can be used to see that when $t = 0$ then the height is 80 feet, meaning the rocket was launched from that high (the top of a building?). The second form is a negative (or zero) number added to 96, which means the maximum height is 96, happening t – 1 = 0 or one second later. The third form shows the height as zero at two possible times, when each factor is zero. Only one of those is positive, namely $t = 1 + \sqrt{6}$ or a bit over 3 seconds later, so that is when the rocket hits the ground. The algebra is similar to what has been asked in the past, converting between different forms of the same function — but just as different forms of fractions are useful for different purposes, the same can be said about different algebraic forms. Switching between algebraic forms is enough work that most people would want to see some payoff from that work. In this case, that means being able to measure the rocket as it is launched and know without further measurements how high it will go and how long it will be in the air.

Purposeful constructions also occur in geometry. In this task (http://www.illustrativemathematics.org/illustrations/508) students are asked to place a fire hydrant at an equal distance from three locations. In class discussion, students can see the purpose for compass and straight-edge constructions which seem arbitrary (and a mouthful to say), like the perpendicular bisectors used here. A teacher gives insight about her experience with the task here: http://easingthehurrysyndrome.wordpress.com/2013/10/14/placing-a-fire-hydrant-2/

In the above two examples, the purpose for some mathematical skills came from a "real-world" context, but purpose is broader than that (so different from our previous discussion). Purposes can include supporting one's own idea about an interesting problem, refuting an alternative idea, or giving a clear mathematical description as in the following task (http://www.illustrativemathematics.org/illustrations/215):

Fred has some colored kitchen floor tiles and wants to choose a pattern using them to make a border around white tiles. He generates patterns by starting with a row of four white tiles. He surrounds these four tiles with a border of colored tiles (Border 1). The design continues as shown below:



Fred writes the expression $4(b-1)+10$ for the number of tiles in each border, where b is the border number, $b \geq 1$.

1. Explain why Fred's expression is correct.
2. Emma wants to start with five tiles in a row. She reasons, "Fred started with four tiles and his expression was $4(b-1)+10$. So if I start with five tiles, the expression will be $5(b-1)+10$." Is Emma's statement correct? Explain your reasoning.
3. If Emma starts with a row of n tiles, what should the expression be?

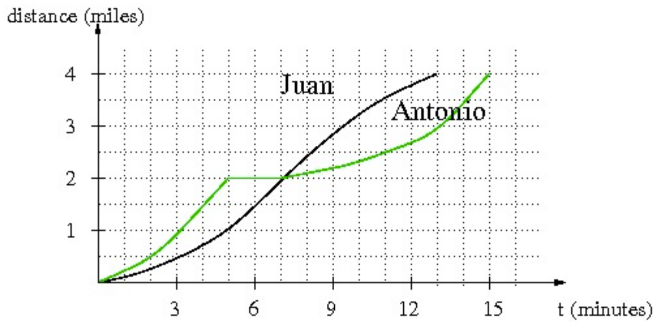
Common Core Shift: Developing meaning, to promote better mathematical skills and application

With new and sometimes greater expectations for what we want students to be able to do, a key question is: "How are we going to help students get there?" One main answer is that we're going to promote full understanding of what they do. Meaning and methods together are the foundation for mastery such as being able to do real-world problems you haven't seen exactly before.

This will start in early grades, for example placing fractions on the number line to understand how they add, as opposed to only memorizing steps to add them. But even in the transition to the Common Core, students can start making sense of mathematics at any time. We've had plenty of positive experience in helping college students understand their elementary math better!

These meanings are often tied in with essential life skills, such as interpreting graphical information. Consider the following task, https://www.illustrativemathematics.org/illustrations/633:

Antonio and Juan are in a 4-mile bike race. The graph below shows the distance of each racer (in miles) as a function of time (in minutes).



1. Who wins the race? How do you know?
2. Imagine you were watching the race and had to announce it over the radio. Write a little story describing the race.

While a great beginning task, it can be a bit tricky even for those who read graphs all the time, since at first it looks like "Antonio goes further." Also, it is a writing task! What a wonderful opportunity for a math teacher and a language arts teacher to team-teach. Finally, think about how we have prepared students to read graphical information in the past — through tasks such as "graph $x^2 - 3x + 4$?" Those have their importance, but it is clear we have been missing tasks like this one that requires students to interpret the information in the graph rather than simply manipulate it.

In the past students were taught to rattle off the basic equation for a line as

$$y = mx + b$$

But what does a linear function *mean*? It means that for every unit one quantity changes, another changes by a fixed amount. For example, for each rubber band added, Barbie is a fixed amount closer to the floor in this activity: http://fawnnguyen.com/barbie-bungee/. Yes, linear functions are given by $y = mx + b$, but that's just part of the meaning. Students need to connect this part of the meaning with other parts (for example, the m in the Barbie activity is the further amount she falls when one rubber band is added) in order to be able to actually use linear functions like we want them to.

What if, instead of adding some fixed amount each time, a quantity gets multiplied each time? That's what an exponential function *means*. Exponential functions have more importance in the Common Core, often appearing earlier than they have in standard curricula because they have a more basic meaning and thus appear in more applications than polynomial functions (traditionally introduced earlier).

Even more basic is the concept of a function and use of function notation, which is at the top of the list — along with some facility with algebra — of what professors need students to know coming into college-level math classes. It takes a lot of time to fully develop the meaning of functions, which can be defined through expressions such as $x^2 + 2$, or 1.2^x, but also as graphs, as tables of (representative) values, as stories (like in the bike task above), or as processes (the value increases by 5 each time). The time taken to understand functions in different ways will pay dividends when students must learn skills such as adding or composing functions. Just as students with better "number sense" will use arithmetic more effectively, students who really understand that linear growth is about constant rates of change will have more "function sense" and be more likely to apply what they know about functions to daily life, college and careers.